INVESTIGATION OF THE SUPERSONIC FLOW AROUND A POINTED ELLIPTICAL CONE AT AN ANGLE OF ATTACK

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The picture of ideal gas flow around cones at zero and low angles of attack has been well studied by using approximate methods [1], and results for high angles of attack have been obtained mainly numerically [2-7]. At high angles of attack it is sensible to examine inviscid flow only up to some generator on the downwind side of the cone at which boundary-layer separation occurs. Hence, the domain where the flow can be considered inviscid yields the main contribution to the magnitude of the aerodynamic forces and the heat fluxes [5, 9]. A picture of the supersonic flow around a pointed elliptical cone is obtained in this paper by the numerical solution of the gasdynamics equations. The whole flow domain is computed at low angles of attack while the solution at high angles is obtained in a domain bounded by some surface of three-dimensional type [10]. The dependence of the flow parameters on the angle of attack is studied when the shock is attached to the cone apex. In contrast to a circular cone, at low angles of attack two spreading lines occur on the surface of an elliptical cone, to which the maximum pressure corresponds. As the angle of attack increases, these lines come together and merge at a certain time. At high angles of attack the flow picture is analogous to a circular cone with a pressure maximum in the plane of symmetry.

1. The nonstationary gasdynamics equations are written in the variables t, n, β under the assumption that the flow is conical just as in [5]. Here t is the time, n is the normalized distance along the normal to the body contour in the cross-sectional plane, and β is the angle between this normal and the plane of flow symmetry. The coordinate β is more preferable as compared with the arc length along the body contour, since the domain with large curvature of the contour, where the gradients of the flow parameters are large, is stretched in this case.

The system of equations is always t-hyperbolic. Hence, finding the stationary conical flow by a time buildup has an advantage compared to a buildup in the axial coordinate [2], which can only be used when the axial velocity component in the shock layer is greater than the speed of sound.

Because of flow symmetry, the problem is solved in half the shock layer domain. The usual symmetry boundary conditions at $\beta = 0$ and π, impermeability on the body, and the conservation condition on the shock are set for the initial system of equations. In the case of the "occlusion" mode, [1], when the conical supersonic flow is closed on the body and the shock is in the downwind zone, the domain of the solution is bounded by some value $\beta_K < \pi$. If the ray $\beta = \beta_K$ is selected so that its corresponding plane $\beta(x, y, z) = \beta_K$ is a surface of three-dimensional type in physical space, then no boundary conditions need be imposed on it [10]. It should be noted that the "occlusion" mode usually holds at high angles of attack when the pressure on the downwind side is low and exerts practically no influence on the aerodynamic characteristics of the cone [5].

The problem is solved numerically by the buildup method, described in detail in the monograph [11]. Formulation of this problem in greater detail and its solution are presented in [12].

2. The conical flow under consideration is symmetric relative to a plane passing through the minor axis of the elliptical cross section. There is a second plane of flow symmetry at a zero angle of attack which

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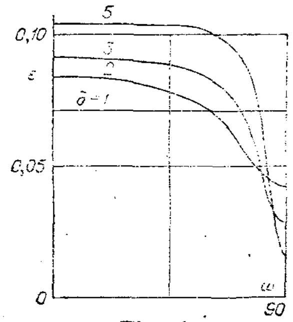

Fig. 1

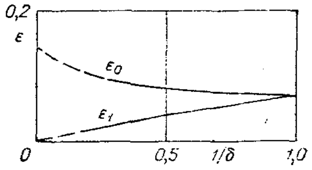

Fig. 2

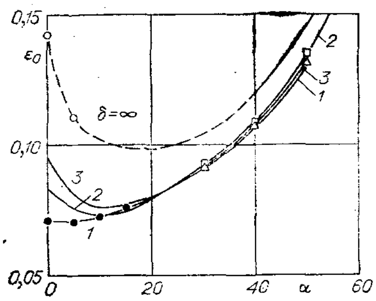

Fig. 3

contains the major axis of the ellipse. In this case, two spreading lines are located on the cone surface in the second plane of symmetry and two runoff lines in the first plane [13]. A conical stream surface with maximum entropy passes through the plane of symmetry, the spreading line, and the cone surface. As the angle of attack increases, both spreading lines are displaced along the cone surface on the upwind side and both runoff lines remain in place. Starting with some angle of attack, the spreading lines merge with the runoff lines in the place of symmetry, and a flow mode with one spreading line from the upwind side and one runoff line from the downwind side is later realized which is analogous to a circular cone. Since it is impossible to predict beforehand which of the two mentioned flow modes will be realized for given parameters, a computation algorithm was produced which would automatically take into account the existence or absence of a runoff line from the upwind side.

The authors had earlier analytically investigated conical flow in the neighborhood of a spreading line located on the body surface [12]. It was shown that a pressure maximum and density maximum correspond to the minimum of the radial velocity component in a spherical coordinate system and that the cone generator along which this holds is a spreading line. Moreover, the conical stream surface is orthogonal to the cone surface along the spreading line.

The flow parameters on the runoff lines where a flow singularity holds were taken different in the computations for the approach along the plane of symmetry and along the body surface, as in [2]. It was assumed that for the approach along the body surface, the entropy equals the entropy on the spreading line whose location has been determined by means of the minimum of the radial velocity component (maximum pressure). In the case with one spreading line, the entropy thereon will automatically be unique and therefore, the remaining flow parameters will also be unique.

The phenomenon of the displacement of the pressure maximum (the spreading line) with the change in angle of attack admits of a graphic interpretation from the viewpoint of Newtonian theory. According to this theory, the pressure coefficient is a maximum where the local angle of attack of the surface is a maximum. Therefore, the spreading line agrees with the generator along which the tangent plane to the cone surface makes the maximum angle with the free-stream velocity vector.

The local angle of attack α' is defined as follows:

$$\sin\alpha' = \frac{(\mathbf{U}_\infty \cdot \operatorname{grad} F)}{|\mathbf{U}_\infty| \cdot |\operatorname{grad} F|}.$$

Finding the extremum of this function in the parameter of an ellipse of cross section ψ, we obtain

$$\cos\psi = \frac{a^2+1}{a^2-b^2}\, b \cdot \operatorname{tg}\alpha, \tag{2.1}$$

where α is the angle of attack, a and b are the major and minor semiaxes of the ellipse in the section $x=1$ and $\psi = \operatorname{arc\,tg}(a/b)\cdot\operatorname{tg}\beta)$. If the expression in the right side is greater than or equal to one, then α' is a maximum in the plane of flow symmetry from the upwind side. As the number M_∞ increases, the Newtonian conception agrees all the more with the numerical solution of the complete gasdynamics equations.

3. Computations of the flow fields of a perfect gas in the shock layer of elliptic cones were carried out. Each variant is characterized by the ratio between the semiaxes of an ellipse of cross section $\delta = a/b$, the semiapex angle of the cone in the plane of the major axis of the ellipse ϑ, the angle of attack α and the free-stream Mach number M_∞. Of considerable interest is the distribution of the conical Mach number M_S which is evaluated according to the velocity component orthogonal to the radius-vector drawn to this point from the cone apex, and the local speed of sound. This Mach number determines the kind of system of stationary gasdynamics equations describing the conical flow, and therefore, the region of influence.

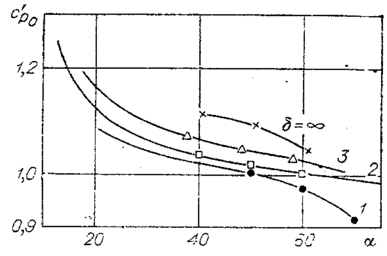

Fig. 4

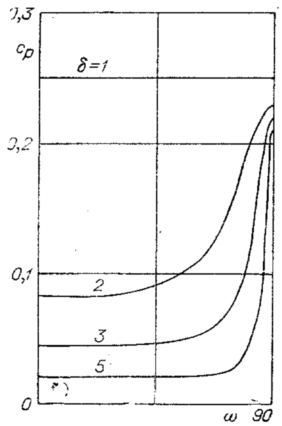

Fig. 5

In [12], the authors earlier presented the lines $M_S = \text{const}$, the isobars and isentropes (lines of intersection of the conical stream surfaces with the cross-sectional plane) for $\delta = 1.788$, $\vartheta = 21.97°$, $M_\infty = 6$, and various angles of attack.

The shock standoff distance $\varepsilon = \varepsilon(\omega)$ in the cross-sectional plane $x = 1$ along the normal to the ellipse contour is presented in Fig. 1. Here $\vartheta = 20°$, $\alpha = 0°$, $M_\infty = 7$; and ω is the meridian angle for this point of the contour, measured from the plane of symmetry from the upwind side. As the minor semiaxis of the ellipse b diminishes, the gradient along the contour grows near the major axis. The shock recedes from the cone surface in the plane of the minor axis of the ellipse and approaches in the plane of the major axis. If the shock standoff $\varepsilon_0 = \varepsilon(0)$ in the plane of the minor axis is constructed as a function of $1/\delta$ (Fig. 2), then as should be expected, ε_0 will tend to its value in the Mach wave as $1/\delta \to 0$ as the elliptical cone tends to a triangular plate. The quantity $\varepsilon_1 = \varepsilon(\pi/2)$ tends to zero in this case (the solid lines correspond to the interval computed). The dependence of ε_0 on the angle of attack α is presented in Fig. 3 ($\vartheta = 20°$, $M_\infty = 7$) for different values of δ. The points here correspond to [2], the triangles and squares to [5], the solid curve for a triangular plate ($\delta = \infty$) is taken from [14], the circles are results of G. P. Voskresenskii, and the Mach wave ($\alpha = 0$). As the angle of attack grows, the quantity ε_0 diminishes, passes through a minimum, and then increases, and the curves for different values of δ approach each other. As δ grows the nonmonotonicity becomes more substantial.

Values of c'_{p_0}, the ratio between the pressure coefficients in the plane of symmetry c_{p_0} and its value calculated by Newtonian theory $2\sin^2\alpha'$ ($\vartheta = 20°$, $M_\infty = 7$), are presented in Fig. 4. Values of the surface local angle of attack α' are plotted along the abscissa. The notation is the same as in Fig. 3, the crosses correspond to [14]. Values of the pressure coefficient for different ellipticities are close to the Newton-theory value in a broad range of angles of attack, with the exception of the domain $\alpha' \leq 20°$. This result can be explained from the viewpoint of [15]. Taking $K_1 = \mathrm{tg}\,\vartheta \,\mathrm{ctg}\,\alpha'$, $K_2 = M_\infty \sin\alpha'$, as has been done in [1], we obtain $K_1 \approx 0.2$ and $K_2 \approx 6$ near $\alpha' = 60°$. The relative pressure coefficient c'_{p_0} for these values of K_1 and K_2 are practically independent of them. For $\alpha \leq 20°$ both parameters are on the order of one.

Distributions of the local pressure coefficient c_p as a function of the meridian angle ω are presented in Fig. 5 ($\vartheta = 20°$, $\alpha = 0°$, $M_\infty = 7$) for different ellipticities. It is seen from the graph that the gradient of c_p near the ellipse major axis grows as the ellipticity increases, and the absolute value of c_p decreases everywhere. As $\delta \to \infty$, the quantity mentioned tends to zero in the plane of the minor axis. The distribution of the pressure coefficient is compared in Fig. 6 with the experimental data in [16]: a) $\delta = 1.788$, $\vartheta = 21.97°$, $M_\infty = 6$; b) $\delta = 1.788$, $\vartheta = 21.97°$, $M_\infty = 3.09$ and in [17]: $\delta = 2$, $\vartheta = 22.5°$, $M_\infty = 3$. The results of the computations agree satisfactorily with the experimental results.

The values of the conical Mach number M_S in Figs. 7 and 8 were taken positive if the velocity component on the cone surface orthogonal to the radius-vector is directed toward increasing angle ω, and negative, otherwise. Therefore, $M_S = 0$ corresponds to the spreading and runoff lines on the graphs, where $dM_S/d\omega > 0$ on the spreading lines and $dM_S/d\omega < 0$ on the runoff lines. At a zero angle of attack (see Fig. 7), the latter holds in the plane of the ellipse major axis and the latter, in the plane of the minor axis ($\vartheta = 20°$, $M_\infty = 7$). As the ellipticity increases, the absolute value of M_S grows, and starting with some values of δ conically supersonic zones ($|M_S| > 1$) already exist at a zero angle of attack.

The displacement of the spreading line ($M_S = 0$, $dM_S/d\omega > 0$) as a function of the angle of attack is seen well in Fig. 8 ($\delta = 1.788$, $\vartheta = 21.97°$, $M_\infty = 6$). For $\alpha \approx 18°$ the spreading line is shifted into the plane of flow symmetry where it remains as the angle of attack increases further. Conically supersonic zones appear at $\alpha = 8°$ [12] in this modification, they reach the body surface ($\alpha = 10°$) as the angle of attack increases, and then the "occlusion" mode is realized.

The dependence of the angle ω', corresponding to the spreading line, on the angle of attack is presented in Fig. 9 for this same cone. A flow mode with one spreading line corresponds to angles of attack lying to the right of the point of intersection of the curve and the horizontal axis. Results obtained by means of (2.1) are superposed by dashes on this graph.

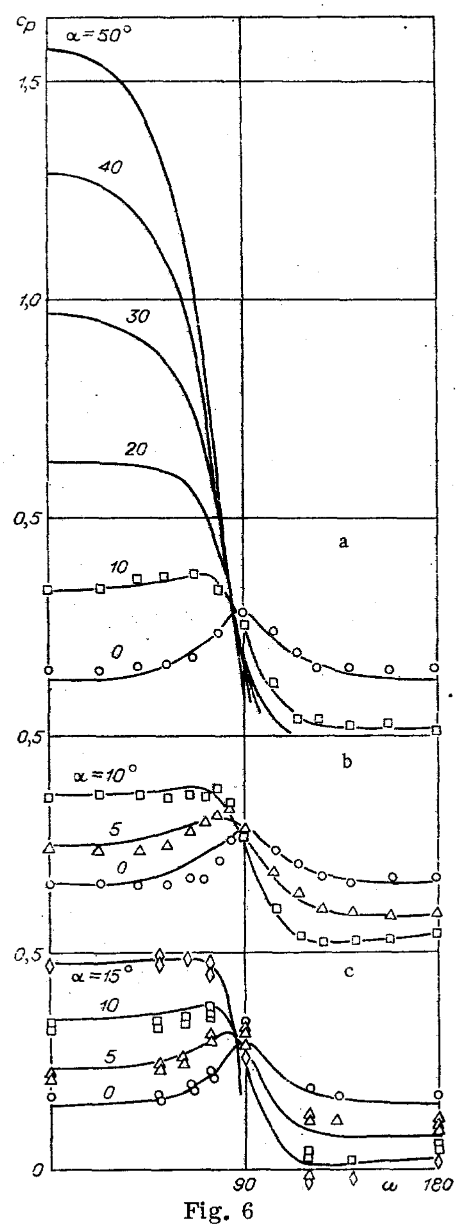

Fig. 6

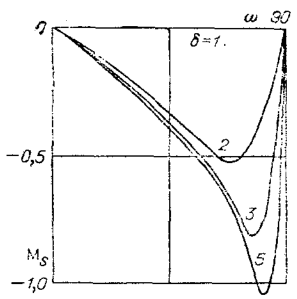

Fig. 7

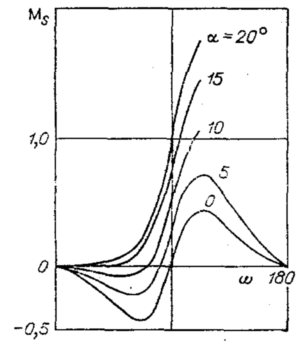

Fig. 8

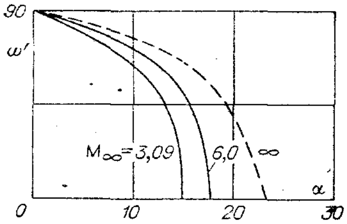

Fig. 9

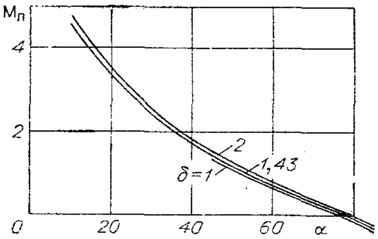

Fig. 10

Formula (2.1) yields a graphical representation of the influence of the elliptic cone parameters and the angle of attack on the position of the spreading line. As δ increases for fixed α, the coordinate $\xi = a \sin \psi$ of the spreading point in the plane of the cone cross section increases, i.e., the point reaches from the plane of flow symmetry. Equating the right side to one, we obtain the value of the angle α^* for which the spreading line is shifted into the plane of symmetry:

$$\operatorname{tg} \alpha^* = \frac{a^2 - b^2}{b(a^2+1)}.$$

It hence follows that the angle α^* increases as the cone flattens more along the minor axis. The error in determining α^* according to this theory apparently grows with the approach to zero. In the other limit case when the cone becomes more circular ($b \to a$), this formula yields the exact result $\alpha^* \to 0$.

The dependence of the total flow Mach number M_{II} on the cone surface is presented for $M_\infty = 7$, $\vartheta = 10°$ in Fig. 10 for $\alpha = 0$. The Mach number at this point is a minimum, as in the case of a circular cone [6]. Starting with some angle of attack, M_{II} becomes less than one. This solution is valid only for an infinite cone or for the neighborhood of the nose in the case of a finite cone. As the angle of attack increases further, M_{II} becomes negative, which corresponds to a change in the flow direction: the stream on the upwind generator starts to flow to the nose. An analogous phenomenon for a narrow triangular plate was first detected by Chernyi [18], and in [6] for a circular cone. It is seen from the graph that as δ increases with the remaining parameters kept fixed, the angles of attack of the passage through $M_{\text{II}} = 1.0$ and $M_{\text{II}} = 0$ grow. This is related to the diminution in the local angle of attack as the cone becomes flatter.

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